Monetary theory

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Chapter 2: Ouput and Money in the short and the long run

Inroduction

- We need to explain the empirical evidence exposed in chapter 1: MP influences prices and output in the short (medium) run but at least for output not in the long run.
- We are going to tackle the impact of MP in a extended IS-LM type of model
- Why extension is needed : IS-LM is a model for the short run : fixed prices, no adjustment of aggregate supply, no revision of expectations, ...
- 2 important features of the model .
- explicit treatment of agents' expectations.
- Model consistent with optimizing schemes : derived from microeconomic behavior which is not explicitly treated here (see Walsh, 5.1 through 5.3).

Basic assumptions

- No physical capital in the model : physical capital does not move $\longrightarrow$ consistent with short-run analysis.
- No trade-off between consumption and leisure : inelastic labor supply.
- Nominal wage rigid during one period : consistent with staggered contracts; nominal wages are fixed during some time and revised only after the end of the first period.

4 Fundamental equations

- Aggregate Supply
- Aggregate Demand
- Money Demand
- Fisher Equation

Aggregate Supply

- $y_t = E_{t-1}y_t + a(p_t - E_{t-1}p_t) + \epsilon_t$
- positive slope $a > 0$ through the price surprise term : based on the standard equilibrium condition: marginal labor productivity equal to real wage.
- $w_t - p_t = y_t - n_t$
- With staggered contracts : $w_t^c = E_{t-1}p_t + E_{t-1}y_t - E_{t-1}n_t$ $\rightarrow$ price innovations (price surprises) decrease ex-post real wage $\rightarrow$ increase output.

Aggregate Demand

- $y_t = E_t y_{t+1} - r_t + u_t$
- How to explain the presence of $E_t y_{t+1}$? Because optimizing agents.
- Intertemporal optimisation between future consumption and interest rates : $y_t = E_t c_{t+1} - r_t = E_t y_{t+1} - r_t + u_t$
- u_t: real demand shock

Money Demand

- $m_t - p_t = y_t - d_2^{-1} i_t + \nu_t$
- Money demand depends on output and interest rate
- In this model, the CB has a direct control on money supply.
- ν_t: money demand shock.

Fisher equation

- $i_t = r_t + E_t p_{t+1} - p_t = r_t + E_t \pi_{t+1}$
- nominal interest rate given by real interest rate and inflation expectations.

Full model

$$y_t = E_{t-1}y_t + a(p_t - E_{t-1}p_t) + \epsilon_t. \tag{1}$$

$$y_t = E_t y_{t+1} - r_t + u_t. \tag{2}$$

$$m_t - p_t = y_t - d_2^{-1} i_t + \nu_t. \tag{3}$$

$$i_t = r_t + E_t p_{t+1} - p_t = r_t + E_t \pi_{t+1}. \tag{4}$$

Comparison with other models

- Wrt IS-LM-AS: the same type of structure but here $E_t y_{t+1}$ in AD : we use the intertemporal optimisation to choose consumption.
- Wrt models in chapter 3: $d_2 = \infty$: elasticity of money demand wrt interest rates zero ; here interest rates play a role.
- key property of the model : an expected increase of money supply will be incorporated in $E_{t-1}p_t \rightarrow$ no surprise $\rightarrow$ no supply side effects.

Model solution

- assume that there is an anticipated increase in money supply m_t .
- in this case , no impact on $p_t - E_{t-1}p_t$, no impact on y_t, no impact on r_t.

Model solution

- $p_t - E_{t-1}p_t = 0 \Leftrightarrow$
- $y_t = \epsilon_t \Leftrightarrow y_t = r_t + u_t$
- $\Leftrightarrow r_t = -\epsilon_t + u_t \Leftrightarrow$
- $m_t - p_t = \epsilon_t - \phi(r_t - E_t p_{t+1} - p_t) + \nu_t \Leftrightarrow$
- $m_t - p_t = \underbrace{\epsilon_t - \phi(-\epsilon_t + u_t)}_{\phi_t} - \phi(E_t p_{t+1} - p_t) \Leftrightarrow$
- $p_t = \frac{m_t + \phi(E_t p_{t+1}) - \phi_t}{(1+\phi)}$

Model solution

$$E_t p_{t+1} = \frac{E_t m_{t+1} + \phi(E_{t+1} p_{t+2}) - \underbrace{E_t \phi_{t+1}}_{0}}{(1+\phi)}$$

$$\Leftrightarrow \quad p_t = \frac{m_t}{(1+\phi)} + \frac{\phi}{(1+\phi)} E_t m_{t+1} + \frac{\phi}{(1+\phi)} E_{t+1} p_{t+2}$$

$$- \frac{1}{(1+\phi)} \phi_t$$

$$\Leftrightarrow \quad p_t = \frac{1}{(1+\phi)} \sum_{i=0}^{\infty} (\frac{\phi}{1+\phi})^i E_t m_{t+i} - \frac{1}{(1+\phi)} \phi_t$$

Model solution

- we see that the price level today depends on the expected flow of future values of the money stock.
- crucial role of expectations . $\longrightarrow$ asymmetric effects of MP depend on whether the increase in m_t is expected or not.

temporary and anticipated MP

- What if increase in m_t is anticipated and temporary ($\rightarrow i = 0$) ?
- p_t increases by $\frac{1}{(1+\phi)} E_t \triangle m_t$
- p_t increases proportionally to $E_t p_{t+1}$
- $m_t - p_t$ increases : one can also see that from Fisher equation : with r_t unchanged, i_t decreases to account for the change in expected inflation $\rightarrow$ increase in demand for real balances

permanent and anticipated MP

- What if increase in m_t is anticipated and permanent ($\rightarrow i = 0, ..., \infty$) ?
- p_t increases by $\frac{1}{(1+\phi)} \sum_{i=0}^{\infty} (\frac{\phi}{1+\phi})^i E_t \triangle m_t$
- $\frac{1}{(1+\phi)} \sum_{i=0}^{\infty} (\frac{\phi}{1+\phi})^i = 1$
- p_t jumps in proportion to $\triangle m_t$
- no impact on real balances $m_t - p_t$, on i_t and on expected inflation.

temporary and not fully anticipated

- What if increase in m_t is not fully anticipated and (temporary) ?
- note : unanticipated and permanent change does not make sense since agents are supposed to be rational.
- p_t increases more than $E_t p_{t-1}$.
- price surprises will affect y_t through supply side effects.
- i_t has to decrease to restore equilibrium in the money market.
- since i_t decreases and in general expected inflation does not decrease , r_t also decreases.
- in turn , real demand will increase $\rightarrow$ matches the increases of the aggregate supply.

Solutions of price and output

- idea : finding analytical solutions of price and output to see (in another way) the influence of monetary policy.
- Combining (2)-(3)-(4) to find an expression of aggregate demand.
- Combining this expression with aggregate supply to have a full equilibrium.

Aggregate demand-output

- Combining (3) and (4) leads to $m_t - p_t = y_t - \phi(r_t + E_t\pi_{t+1}) + \nu_t$.
- using (2): $r_t = E_t y_{t+1} - y_t + u_t$.
- $\Leftrightarrow y_t + \phi y_t = m_t - p_t + \phi E_t y_{t+1} + \phi E_t \pi_{t+1} + \phi u_t + \nu_t$
- $\Leftrightarrow y_t = \frac{d_2(m_t - p_t) + E_t y_{t+1} + E_t \pi_{t+1} + u_t - d_2 \nu_t}{1 + d_2}$
- We have a standard aggregate demand equation with negative relationship between y_t and p_t.
- with m_t affecting aggregate demand.
- inflation expectations affect y_t.

Aggregate demand-price

- We need to find the equilibrium of AD and AS and find the equilibrium price level.
- AS=AD

$$E_{t-1}y_t + ap_t - aE_{t-1}p_t + \epsilon_t = E_t y_{t+1} - r_t + u_t$$

- $\Rightarrow$

$$p_t = \frac{d_2 m_t + a(1+d_2)E_{t-1}p_{t-1} + E_t\pi_{t+1} + u_t}{d_2 + a(1+d_2)}$$

$$\frac{-d_2\nu_t - (1+d_2)\epsilon_t}{d_2 + a(1+d_2)}.$$

- We need to get rid of $E_{t-1}p_t$ and express everything in terms of $p_t - E_{t-1}p_t$ because this term plays a crucial role for explaining y_t.

Equilibrium

$$p_t - E_{t-1}p_t = \frac{d_2(m_t - E_{t-1}m_t) + (E_t\pi_{t+1} - E_{t-1}\pi_{t+1})}{d_2 + a(1+d_2)} +$$

$$\frac{u_t - d_2\nu_t - (1+d_2)\epsilon_t}{d_2 + a(1+d_2)}.$$

- we can substitute this surprise term into the AS equation to get the equilibrium value of y_t consistent with both the supply and the demand
-

$$y_t = \frac{a[d_2(m_t - E_{t-1}m_t) + (E_t\pi_{t+1} - E_{t-1}\pi_{t+1})]}{d_2 + a(1+d_2)} +$$

$$\frac{a[u_t - d_2\nu_t - (1+d_2)\epsilon_t]}{d_2 + a(1+d_2)}.$$

Comments

- This confirms that an unanticipated increase of m_t increases y_t .
- On the supply side, $p_t - E_{t-1}p_t > 0$ decreases real wages $\longrightarrow$ employment increases along with output.
- On the demand side, r_t decreases; i_t also decreases since $E_{t+1}p_{t+1} - p_t$ decreases and $m_t - p_t$ increases.

Treatment of $E_t\pi_{t+1}$

- Up to now, we have assumed that $E_t\pi_{t+1}$ does not change as a result of an increase in $m_t \rightarrow$ strong assumption.
- what if $(m_t - E_{t-1}m_t)$ and $E_t\pi_{t+1}$ both increase ?
- we see from the surprise equation that $p_t - E_{t-1}p_t$ will increase more $\rightarrow$ the impact on y_t will be more important.
- we see also that the increase in $m_t - p_t$ will be less important. Why ? The increase in $E_t\pi_{t+1}$ will induce a less important decrease in i_t (see Fisher equation).

Treatment of $E_t\pi_{t+1}$

- Let us express $E_t\pi_{t+1}$ as $E_tp_{t+1} - p_t$.
- we obtain :

$$p_t = \frac{d_2m_t + E_tp_{t+1} + a(1+d_2)E_{t-1}p_t}{(1+a)(1+d_2)} + \frac{u_t - d_2\nu_t - (1+d_2)\epsilon_t}{(1+a)(1+d_2)}.$$

- We see that since p_t depends on m_t, $E_{t-1}p_t$ depends on $E_{t-1}m_t$, and E_tp_{t+1} depends on E_tm_{t+1} $\rightarrow$ we need to see how expectations of m_t are built.
- To this aim, we are going to assume a particular process of money supply that the agents can use to build their expectations.

Money supply process

- assume the following process for m_t :

$$m_t = \mu + m_{t-1} + a_1 u_t + a_2 \epsilon_t + a_3 \nu_t + \omega_t.$$

- μ: average growth rate of m_t.
- ω_t: control error of the central bank : the CB does not control perfectly the monetary creation process.
- a_1,a_2,a_3: coefficients of reaction to supply shocks, demand shocks and money demand shocks.
- use that to compute $E_{t-1}m_t$ and hence $E_{t-1}p_t$.

Solution (not explained)

- $$p_t = \frac{\mu(1+d_2)}{d_2} + m_{t-1} + b_1 u_t + b_2 \epsilon_t + b_3 \nu_t + b_4 \omega_t.$$

- $b_1 = (1 + a_1(1 + d_2))$;$b_2 = \frac{a_2 - 1}{1+a}$;
  $b_3 = (a_3(1 + d_2) - d_2)$;$b_4 = \frac{1}{1+a}$

- $$y_t = \frac{(1 + a_1(1+d_2))u_t + (a_3(1+d_2) - d_2)\nu_t + (1+d_2)\omega_t}{(1+a)(1+d_2)}$$

  $$+ \frac{(aa_2 + 1)\epsilon_t}{1+a}.$$

Implications

- We see that output does not depend on μ and m_{t-1}
- We see that output depends on the a_i parameters .
- These parameters capture the way monetary policy is conducted, i.e. m_t reacts to the occurrence of shocks.
- This illustrates the Lucas critique : the impact of monetary policy depends on the way monetary policy is conducted.
- Here we assume that the CB reacts directly to the occurrence of shocks.

Examples

- $a_1 < 0$: a positive demand shock leads to a restrictive monetary policy. This dampens the positive direct effect of u_t on y_t.
- $a_2 = -\frac{1}{a} < 0 \rightarrow$ output is isolated from supply shocks and the impact on prices is amplified.
- $a_2 = 1 > 0 \rightarrow b_2 = 0 \rightarrow$ no impact of supply shocks on prices and effect on y_t amplified .
- $a_1 = \frac{-1}{1+d_2}$, $a_2 = 1$, $a_3 = \frac{d_2}{1+d_2} \rightarrow$ prices become independent from shocks.
- the way the CB reacts to shocks has an impact on the effect of shocks on prices and output.

Inflation expectations

- $$E_t\pi_{t+1} = \mu + (a_1 - b_1)u_t + (a_2 - b_2)\epsilon_t + (a_3 - b_3)\nu_t + (1 - b_4)\omega_t.$$
- The expected value of inflation depends on the average growth rate of the money stock.
- on the shocks.
- on the discrepancy between the parameters of the CB reaction function (a_i's) and the structural parameters (b_i's).

Interest rate: equilibrium

- $$i_t = d_2(y_t - m_t + p_t + \nu_t).$$
  $$i_t = \mu + \frac{d_2}{1+d_2}(u_t - d_2\nu_t).$$
- the equilibrium interest rate does not depend on ω_t. Why ?
- The shocks to m_t are permanent; assume $\omega > 0 \Rightarrow p$ increases by $\frac{\omega}{1+a}$.
- p also increases next period and surprise inflation increases y_t.
- This requires an increase in real demand through a decrease in r_t.
- i_t remains unchanged because r_t decreases and $E_t\pi_{t+1}$ increases.

Interest rate: equilibrium

- In the long run, we see that i_t depends on μ
- In the long run, $E_t\pi_{t+1}$ and π_t depend on μ .
- The model is able to reproduce the impact of m_t in the short run : positive on output and prices.
- The model is able to reproduce the impact of m_t in the long run run : no impact on output and positive on prices.

Interest rate policy

- What if i_t used as the MP instrument rather than m_t ?
- This is the case today if we see the way MP is implemented $\rightarrow$ major shift observed in the beginning of the 80's (see chapter 4 for rationalization)
- rewrite AS in terms of inflation rather than price level:

$$y_t = E_{t-1}y_t + a(\pi_t - E_{t-1}\pi_t) + \epsilon_t$$

- We can substitute Fisher equation in AD equation to get:

$$y_t = E_t y_{t+1} - (i_t - E_t\pi_{t+1}) + u_t$$

- We can find the solution in terms of p_t and y_t from these two equations.

Interest rate policy

- The general level of prices is not expressed here but rather inflation $\rightarrow$ issue of indeterminacy of the price level .
- The money demand equation is no longer necessary to find the solution $\rightarrow$ no role for money demand. Why ?
- When using interest rates, monetary authorities adjust i_t and m_t adjusts to find the money equilibrium (given y_t).
- We need to assume a particular monetary rule using interest rate.To this aim, we choose the famous Taylor rule .

Taylor Rule

- describes the behaviour of the CB in terms of MP
- $i_t = b_y y_t + b_\pi \pi_t$
- Taylor has proposed the following values : $b_y = 0.5$ and $b_\pi = 1.5$. Implies a restrictive monetary policy when business cycle is relatively high and inflation relatively high.
- This rule fits the data rather well: reproduces the dynamics of the prime rates decided by the Fed over the 15 previous years.
- Example: Taylor

Solutions

- replace the implied value of i_t in the AD equation:

$$y_t = \frac{E_t y_{t+1} - (b_\pi \pi_t - E_t \pi_{t+1}) + u_t}{1 + b_y}$$

  .

- We assume $E_t y_{t+1} = 0 = E_{t-1} y_t$
- we can look for the values of π_t and y_t that equate AD and AS

Solutions

- $$\pi_t = \frac{u_t - (1+b_y)\epsilon_t}{a(1+b_y)+b_\pi}.$$
  $$y_t = \frac{au_t + b_\pi \epsilon_t}{a(1+b_y)+b_\pi}.$$
- The MP parameters once more play a role in determining the impact of MP $\rightarrow$ consistent with Lucas critique .
- Role of shocks is emphasized
- By constrast, no role for money demand shocks because these are fully absorbed by the interest rate policy $\rightarrow$ more on this in chapter 4.